vortex model, whether the rollup of lift-generated vortex sheets can be suppressed. He has predicted a substantial reduction in rolling moment for wings that encounter the wakes that trail from span loadings with a sawtooth character. The basic premise of the proposal is that wakes comprising many counter-rotating vortices are more apt to produce erratic motions which diffuse the vorticity than a single pair of stronger vortices.

An essential feature of the discrete-vortex model is that the larger the number of the point vortices and the vortex sheets the larger the error growth during the computation. Rossow and the references cited by him discussed the shortcomings of the method. Similar computational difficulties in the use of the discrete-vortex model have been encountered by this writer[2,3] and resolved through the use of various techniques which help to suppress the vortex excursions and sheet kinking. Such techniques, as pointed out by Rossow, contain arbitrary parameters that are not related to the conservation equations for the fluid they are to represent. The purpose of this Note is to point out that *the primary source of difficulty is in not rediscreticising the sheet at each time interval of the calculation* and that it is not possible to calculate the motion of a vortex sheet from the motion of the series of discrete vortices used to represent that sheet for more than infinitesimal time intervals.

Fink and Soh,[4] in a report published about six months prior to the receipt of the revised version of Rossow's paper,[1] have shown that the complex conjugate velocity $\bar{q}(z_j)$ at a point z_j, on the vortex sheet, due to series of n discrete vortices is given by

$$\bar{q}(z_j) = \frac{1}{2i\pi} \sum_{k\neq j}^{n} \frac{r_k}{z_j - z_k} - \frac{1}{2i\pi} \frac{\Gamma_{je}{}^{-i\theta}j}{|s_{j+1/2} - s_{j-1/2}|} Ln \left| \frac{z_j - z_{j+1/2}}{z_j - z_{j-1/2}} \right|$$

The point z_j *lies with the segment* $(s_{j+1/2}, s_{j-1/2})$, without necessarily bisecting it at all times, and s measures the distance along the sheet. The remarkable consequences of this expression are that: a) if the equivalent vortex is not placed at the mid-point of its segment through rediscretization of the sheet at each time interval, then the logarithmic term does not vanish and the computational error increases depending on the problem, the number of vortices and time interval used, and the total time of computation; b) the vortices which initially bisect the segment which they are to represent do not continue to do so at the succeeding time intervals; c) the use of finite vortex cores, accumulation of vortices at the center of the spiral, or other techniques only delay or minimize the accumulation of the errors resulting from the logarithmic term in an amount related to the distance between z_j and the center of the segment; and that d) the growth of the computational error may be significantly reduced by placing each discrete vortex at the mid-point of its segment, i.e. by placing the vortex at $z_j = 0.5(z_{j-1/2} + z_{j+1/2})$ at each time interval. Only through such a procedure that one can make the logarithmic term vanish.

The calculations are then carried out at each time step by representing the vorticity density by an entirely new set of equi-distant vortices whose strengths are adjusted to give a good representation of that density. Evidently, this procedure does not resolve all of the computational errors particularly in regions where the curvature of the sheet is small, e.g., the region close to the center of the vortex spiral. Furthermore, the curve-fitting errors incurred in the process of interpolation at every time step may accumulate as time increases. Nevertheless, Fink and Soh[4] have shown through several examples that the rediscretization method overcomes many of the difficulties encountered by the previous users of the discrete-vortex model.

This writer has applied the rediscretization or the resegmentation method to the solution of the flow past an inclined plate (already worked out by him[3] through the use of the discrete vortex model without rediscretization) and found that: a) rediscretization could be used only for relatively small times ($Ut/s < 1$); b) the forces acting on the plate predicted without without rediscretization are practically identical; c) the decreasing separation between the turns of the spiralling sheet causes interaction and orbiting between the adjacent vortices and requires segment lengths smaller than the separation distance. This, in turn, considerably increases the computation time even for $Ut/s < 1$.

Evidently, rediscretization can be effective under certain circumstances. It is not, however, ready to replace the current techniques of application of the discrete-vortex model. Considering all other alternatives for the calculation of separated flows for which the wake is not replaced by a dead body of fluid, the discrete vortex approximation can produce numerically stable and viable solutions through the use of judiciously selected parameters such as the number of vortices, time interval during which the vortices are convected, etc. Rossow, like many others who have used this method, seems to have struck such a balance in his exploratory calculations of a rather complex problem.

The idea of tailored loading is an interesting one and may lead to the reduction of wake-vortex hazard. As such, it deserves further analysis with evenly as well as unevenly spaced sawtooth configurations. However, it remains to be seen whether the tailored loading will prove to be more practical and superior to other methods proposed such as the use of a 2500 HP engine at each wing tip of a DC-10 which is to prevent, through its counter rotating flow, the formation of the concentrated vortices without affecting the lift-to-drag ratio. Model studies based on this concept are currently being carried out by the Garrett Corp. at Pheonix, Arizona.[5]

References

[1] Rossow, V. J., "Theoretical Study of Lift-Generated Vortex Wakes Designed to Avoid Rollup," *AIAA Journal*, Vol. 13, April 1975, pp. 476-484.

[2] Sarpkaya, T., "An Analytical Study of Separated Flow About Circular Cylinders," *Journal of Basic Engineering*, Transactions of the ASME, Vol. D-90, No. 4, Dec. 1968, pp. 511-520.

[3] Sarpkaya, T., "An Inviscid Model of Two-Dimensional Vortex Shedding for Transient and Asymptotically Steady Separated Flow Over an Inclined Plate," *Journal of Fluid Mechanics*, Vol. 68, Pt. 1, March 11, 1975, pp. 109-128

[4] Fink, P. T. and Soh, W. K., "Calculation of Vortex Sheets in Unsteady Flow and Applications in Ship Hydrodynamics," Rept. NAV/ARCH 74/1, School of Mechanical and Industrial Engineering, The University of New South Wales, Australia, April 1974.

[5] Personal communication with Mr. J. R. Erwin, Airesearch Manufacturing Company of Arizona, A division of Garrett Corporation, Sky Harbor, Phoenix, Ariz., June 1974.

Reply by Author to T.Sarpkaya

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THE Technical comment by T. Sarpkaya calls attention to a new method derived by Fink and Soh[1] for predicting the rollup of vortex sheets. This method differs from others in that it contains a logarithmic term in the ex-

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pression for the velocity of the discrete vortices that are used to represent the sheet. I have not compared the results obtained by the Fink-Soh method with those obtained by the simple discrete-vortex method and, therefore, am not able to evaluate or comment on the virtues of their technique. The purpose of this Reply is to point out that their method applies only to those cases where rollup of a continuous vortex sheet is being analyzed, e.g., the tailored and elliptic loading cases presented in Ref. 2. Such a method does then not apply to systems of vortices in general nor to vortex wakes that are composed of discrete vortices such as the stepped and sawtooth loadings in my paper.[2]

Based on the experience with the method of Fink and Soh that Prof. Sarpkaya reported in his Comment, it appears unlikely that inclusion of the logarithmic term in the velocity expression would significantly alter the results presented in Ref. 2 for tailored and elliptic loadings. Also, as noted in Ref. 2, the numerical calculations were monitored with accuracy parameters and then terminated to prevent error growth beyond a specified limit. The resulting wake vortex structure was then found to be in good agreement with Betz' rollup theory (Figs. 6 and 10 of Ref. 2). Therefore, all of the numerical results remain correct as presented in Ref. 2, and the conclusions drawn in the paper need not be modified.

As pointed out in Ref. 2, the erratic character and large magnitude of the excursions predicted for the vortices shed by stepped and sawtooth loadings has been confirmed qualitatively by experiments conducted at NASA Ames Research Center (e.g. Ciffone and Orloff[3]). It still remains however, to apply these loading concepts effectively so that the wake-dispersive motions of the vortices bring about adequate alleviation of the wake hazard behind large aircraft, as well as to compare the alleviation achieved and penalties incurred by the various aerodynamic-alleviation schemes with vortex avoidance systems.

References

[1]Fink, P.T. and Soh, W.K., "Calculation of Vortex Sheets in Unsteady Flow and Applications in Ship Hydrodynamics," Rept. NAV/ARCH 74/1, School of Mechanical and Industrial Engineering, The University of New South Wales, Australia, April 1974.

[2]Rossow, V.J., "Theoretical Study of Lift-Generated Vortex Wakes Designed to Avoid Rollup," *AIAA Journal*, Vol. 13, April 1975, pp. 476-484.

[3]Ciffone, D.L. and Orloff, K.L., "Far-Field Wake-Vortex Characteristics of Wings," *Journal of Aircraft*, Vol. 12, May 1975, pp. 464-470.

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